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The New Similarity Rules Applied to Argon Microwave Noise Sources

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***Abstract*—It is shown that when the noise temperatures of argon plasma noise generators, operated at fixed current/radius ratios, are plotted as $1/T_N$ versus ln (pr), the experimental data form a straight line.**

Introduction

The purposes of this paper are to show that the noise temperatures of commercial argon noise sources agree when a comparison is made based upon the new similarity rules which require scaling at constant current/radius ratios, and to show that the data obey a relationship of the form $1/T_N \propto \ln (pr)$, which permits using a linear least squares fit of experimental data.

Historical Microwave Developments

Beginning with the invention of the plasma noise source by Mumford in 1949 [1], many workers measured the noise temperatures of plasmas. An extensive experimental investigation and summary of previously published data was made by Olson in 1968 [2]. His conclusions were that the noise temperatures measured under various conditions did not satisfactorily agree, and that they did not agree with the von Engel and Steenbeck theoretical value [3, p. 86], [4, p. 242].

Later, it was discovered that a close agreement appeared to exist between his data and that taken by Denson and Halford [5] when only data from wall-contained plasmas were considered; this was shown in [6, fig. 1]. Comparisons between the noise temperatures obtained from different radius tubes were *always* made by invoking the traditional similarity rules [4, p. 288], [7, p. 209], [8, p. 59] which required that the current be the same in all tubes, independent of the radii.

The New Similarity Rules

In 1969 a new set of similarity laws requiring scaling at constant current/radius ratios was formulated by Pfau *et al.* [9]. Unfortunately, the new rules were not immediately widely known, and they were "rediscovered" at least twice since then. In 1975 it was shown in [10, fig. 4] that the new

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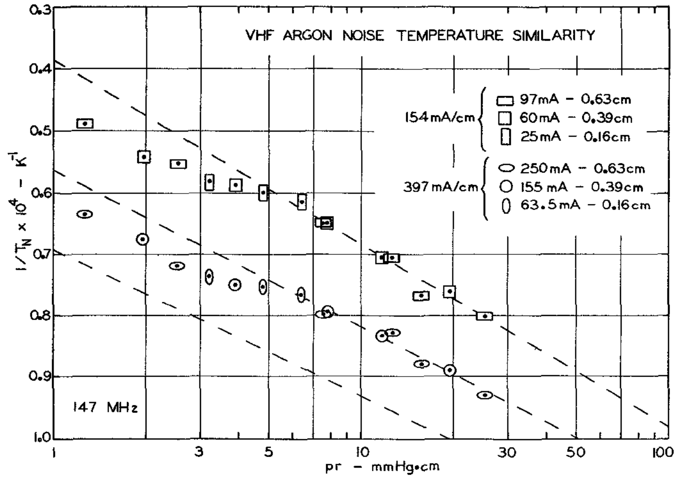

Fig. 1. The reciprocal of the VHF argon plasma noise temperature as a function of the log of the *pr* product, with the discharge currents scaled at fixed current/radius ratios of 154 mA/cm for the upper data and 397 mA/cm for the lower. The data form straight lines because of the method of plotting.

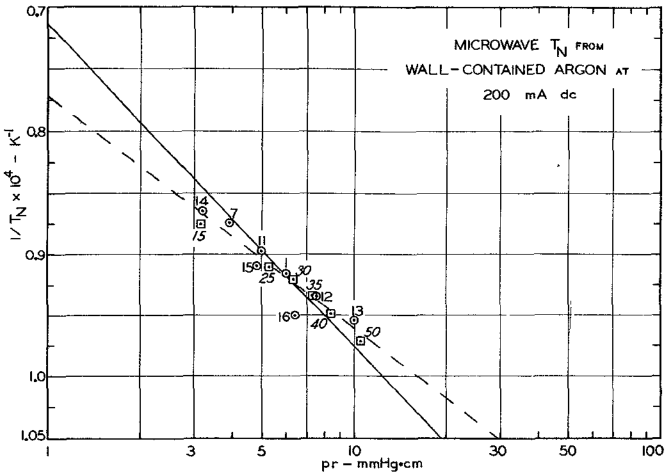

Fig. 2. The reciprocal of the measured microwave noise temperature T_N from wall-contained Ar discharges at 200 mA dc versus the log of the *pr* product. Dashed line is a linear least squares fit of the data. Solid line and data points are the same as in [6, fig. 1].

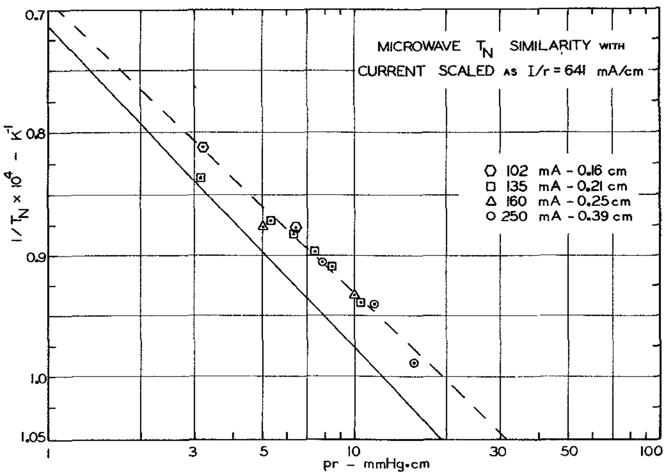

Fig. 3. The reciprocal of the measured microwave noise temperature T_N versus the log of the *pr* product. Data points were taken from Olson and from Denson and Halford at a fixed current/radius ratio of 641 mA/cm. Dashed line is a linear least-squares fit of the data; solid line is as in Fig. 2.

similarity laws resulted in a remarkable correlation between VHF noise temperatures obtained from tubes with radius ratios of 4:1, thus finally solving a problem closely analogous to the microwave noise temperature situation.

Linearizing The VHF Data

A problem remaining with the VHF data was that the curves they generated had unspecified shapes. Thus only a polynomial least squares technique could be used to generate a curve from the data, and it was not possible to determine whether the curve was "correct."

Subsequently, it was discovered that when the reciprocal of the noise temperature was plotted as a function of the log of the *pr* (pressure–radius) product, straight lines were formed; the VHF noise temperatures given in [10, fig. 4] are replotted on this basis in Fig. 1.

The method of plotting reveals a pronounced break in the data at $pr \cong 4$ mmHg · cm which corresponds with the longitudinal electric field minimum, and the boundary between wall-contained and constricted positive columns—this break is totally hidden in the traditional T_N versus *pr* plots; e.g., [10, fig. 4].

The ability to plot the data on a linear basis provides a simple means to generate a "curve," and indicates that the data obey a relationship of the form

$$\exp\,(qV_i/kT_N) = c_2(pr)^n \tag{1}$$

where c_2 is a constant and $n \cong 2$. This formula is similar to the traditional von Engel and Steenbeck formula [3, p. 86], [4, p. 242].

The dashed lines are linear least squares fits of the data for $pr \geq 5$ mmHg · cm, giving a value of $n = 2.36$ in (1) for the upper (154 mA/cm) data, and $n = 2.04$ for the lower (397 mA/cm) data. The lowest dashed line is a least squares fit for the microwave data at 641 mA/cm (as given in Fig. 3).

A New Look at the Microwave Data

Armed with the knowledge that a plot of the reciprocal of noise temperature as a function of the log of the *pr* product should form a straight line, the data given in [6, fig. 1] were replotted on this basis as shown in Fig. 2. A linear least squares fit of the data is indicated by the dashed line; the solid line is the von Engel and Steebeck formula with $c = 0.106$. The data are all for a discharge current of 200 mA dc.

Although the data do form a consistent set, the new similarity rules require scaling the current at a constant current/radius ratio instead of holding the current constant as was done in Fig. 2. Also, the slope of the dashed line gives a value of $n = 1.5$ in (1); from the experience gained with the VHF noise temperature measurements, the slope is expected to be about 2.

Therefore, a new set of points was chosen[1] from the data published by Olson [2] and by Denson and Halford [5], with the current/radius ratio held constant at 641 mA/cm. These data are given in Fig. 3. The dashed line represents a linear least squares fit of the data: $n = 1.9$ in (1). The solid line is the von Engel and Steenbeck formula with $c = 0.106$. Not only do the data form a consistent set, but also the slope is much closer to the expected value than it was in Fig. 2.

Conclusion

It has been shown that plasma noise sources obey the new similarity rules which require scaling the currents at constant current/radius ratios, thus finally resolving the apparent inconsistencies in the measurements taken over the last 25 years. Also, the data can be plotted as a straight line which permits easy generation of a curve from the data and easy identification of deviations in the data.

[1] In spite of the huge volume of published data on plasma noise temperatures, the vast majority of it cannot be used for various reasons: 1) The radius is not specified. 2) The current dependence is not given. 3) The filling pressure is not given. 4) The currents used are outside of the I/r range available from other tubes. Probably the reasons for the above state of affairs are as follows. 1) The traditional similarity rules were adhered to too closely; thus data were usually given for only one current, and only the pr product was given, not p or r. 2) Overzealous editors demanded omission of some of the data for sake of brevity; it seems that all unpublished portions of data "vaporize" shortly after the published portions actually appear in print. Therefore, only at an I/r ratio of 641 mA/cm was enough microwave data available to present enough points to establish a curve.

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Letters

Interdependence of Gain and Idler Conversion Loss in Parametric Amplifiers

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***Abstract*—Interdependence of gain and idler conversion has been experimentally investigated in parametric amplifiers having single diode circuit configuration. The results indicate that for high gain amplifiers, the inherent idler rejection is inadequate, and larger lengths of pump waveguide, designed to be below cutoff at idler frequency, are to be used for reducing the effect of pump sideband noise on the noise temperature of the parametric amplifiers.**

I. Introduction

The subject of pump noise transfer in parametric amplifiers has recently received considerable attention [1]–[3]. The noise sidebands associated with the pump source can significantly degrade the noise figure of the parametric amplifiers. Tearle and Heath [3], have shown that the degradation in noise temperature due to AM noise associated with the pump source can be predicted to a fair degree of accuracy with the expression

$$10 \log \Delta Te = \frac{N}{C} + Po - A - 10 \log k - G \qquad (1)$$

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where ΔTe is the degradation in noise temperature, N/C is the AM noise of the pump source, Po is the pump power, A is the conversion loss, and G is the gain of the amplifier. The conversion loss A is the ratio of the injected power at the idler frequency to the output power at the signal frequency. Expression (1) shows that the degradation in noise temperature can be made very small by increasing the conversion loss A of the amplifier.

The idler conversion loss A, as conceived by Tearle and Heath [3], is made up of two parts:

1) that which determines the amount of power at idler frequency coupled into the idler circuit;
2) that which determines the amount of idler power down-converted to power at signal frequency.

The contribution due to part 1) depends on the circuit configuration and is different for single-diode and double-diode parametric amplifiers. In the case of the double-diode circuits, part 1) can be adjusted independently of paramp gain to a very high value either by making small adjustments to varactor dc bias voltages or by preselection of well-matched diode pairs [3]. As a consequence of this, very little power at idler frequency (e.g., sideband noise on the pump) is coupled into the idler circuit, and the degradation in noise temperature due to AM